Assessing Robustness of Planning Level Tools for Predicting Roundabout Behavior

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ABSTRACT

Delay is considered as the primary measure of effectiveness for both signalized and un-signalized intersections according to the Highway Capacity Manual (HCM). However, the HCM method for calculating delay at roundabouts is complex to the point of requiring computer software to compute, which is impractical for purposes of preliminary design engineering. This research seeks to determine if a simplified methodology, the critical sum method, is sufficient for predicting the anticipated average control delay of a single-lane roundabout.

With a simple mental calculation, the critical sum method can provide an indication of performance of a facility at a preliminary stage of design, but the reliability of this prediction when translated to a delay measure is not well-documented. The researchers, using the algorithmic formulations for both the critical sum method and the HCM method, ran 250,000 volume scenarios, systematically varying parameter combinations. Although a consistent relationship between the critical sum value and the average vehicle delay was determined, ultimately the reliability of this relationship within critical volume ranges was found to be insufficient. The critical sum value proved to be a reliable indicator variable for average delay only when average delay was below 15 seconds per vehicle. Volume scenario factors such as directional split and turn movement percentages had minimal effect on these findings.

BACKGROUND

In the 1960's, the United Kingdom resolved the safety issues associated with traffic circles by effecting yield control on vehicles entering the facility and decreasing the inscribed diameter of the circle to reduce circulating speeds. Today this geometric configuration, widely known as a "modern roundabout," is common place, and a standard solution for engineers to consider. Several common problems with conventional signalized intersections can be resolved with the use of a modern roundabout, including: enhance safety, speed reduction enforcement, crash frequency and severity, and minimizing traffic delays could be resolved with roundabouts.[1]–[6]

Generally, the performance analysis of an intersection yields two key pieces of information: capacity and average delay, with secondary parameters of importance like maximum vehicle queue. When calculating a facility's Level of Service (LOS), the Highway Capacity Manual utilizes delay as the primary measure of effectiveness for both signalized and unsignalized intersections. [7]

Moreover, only the control delay is considered in the HCM methodology, expressly excluding changes in travel distance caused by the facility. Control delay is defined as the time a driver is involved in various queuing situations, such as deceleration upon approach, entering, exiting, and wait time for an acceptable gap at the front of the queue when entering circulating flow. [8] For a single-lane roundabout, the HCM method calculates the average control delay using a logarithmic mathematical equation, utilizing lane capacity, volume to capacity ratio, and analysis period as the input parameters.

Current practice for roundabout operations focuses on identifying individual approach capacity as a function of circulating and entering flow. While geometric elements can impact the overall flowrate of an approach, the circulating flow is the primary constraint. The circulating flow directly impacts the availability of adequate gaps in the roundabout, with low flow equating to more gaps, allowing drivers to enter without significant delay. [7] As circulating flow increases the size of the gaps decrease, leading to decreases in capacity.

In modern roundabouts, gap acceptance is generally modeled as being directly influenced by two parameters, the critical headway and the follow-up headway. Critical headway is measured on the circulating lane, and is the minimum time between two successive circulating vehicles required by a driver to safely enter the roundabout between those vehicles. The National Cooperative Highway Research Program (NCHRP) Report 572 defines critical headway as greater than the largest headway rejected and shorter than the accepted headway. [9] Critical headway has also been found to be influenced by local conditions such as: driver behavior, geometric layout, and prevailing traffic conditions. [10] Follow-up headway is measured on the approach lane, and is defined as the minimum time between two successive entering vehicles entering a roundabout into the same circulating gap. The follow-up headway is estimated by taking the average difference between the passage times of two entering vehicles accepting a gap in the conflicting stream under a queued condition. [11]

There are several methodologies for calculating the volume-to-capacity ratio of roundabout approaches. The critical sum method uses the most basic calculations of the methods discussed here, providing a measure of demand by adding the volume of a given approach to the volume of the circulating flow that opposes that approach. The capacity against which this demand is compared could be calibrated by situation or geometry, but is generally taken by default as equal to 1,600 veh/h/ln independent of other factors, which represents the total number of vehicles in conflict with each other that can be serviced by a facility. [12] By contrast, the current 6th edition of the Highway Capacity Manual (HCM) calculates the maximum capacity of a given approach using an exponential regression model based on gap acceptance, determining the volume-to-capacity ratio based on the demand flow rate of the approach. [8] A popular software application from Australia, Signalized and Un-signalized Intersection Design and Research Aid (SIDRA), calibrates gap acceptance incorporating geometric elements with demand flows. [13], [14] In terms of complexity, the most comprehensive modeling of roundabout performance comes from the United Kingdom, where ROundabout DELay (RODEL) calibrates total entry capacity per approach making extensive use of the geometric design of the facility. [15], [16] Although it is also a popular choice domestically for validating roundabout performance predictions, some studies indicated the failure of the RODEL model in estimating the capacity at roundabouts with congested conditions. [17]–[19]

OBJECTIVE AND PURPOSE

The main scope of this paper is to explore the degree to which the critical sum method can provide a planning-level assessment of roundabout performance that will be consistent with the results of more formal analysis using HCM delay procedures. The intention of the authors is to assess this relationship for a relatively simple single-lane roundabout configuration, with further study into larger facilities should the initial investigation provide positive results. In assessing the effectiveness of the critical sum method as a planning tool for preliminary engineering, the following objectives are identified:

1. Identify a best-fit relationship between the critical sum value and the HCM delay for a single-lane roundabout facility.
2. Indicate the reliability of the above relationship for practical use. How does it progress when congestion increases, and up to what volume can it be considered an effective indicator of delay?
3. Identify if the nature of the volume scenario has an impact on the reliability of the critical sum method as an indicator variable for delay.

EVALUATION APPROACH

The authors are interested in investigating the predicted average control delay based upon a given critical sum value. Additionally, the question of whether the predicted output can be practically implemented or reliable for use is also a vital concern. As a measure of reliability, the authors will indicate the percentage of the data points with a delay with in an acceptable range from the actual mean delay. This enables the authors to indicate and document the maximum range/limit to which the critical sum method is recommended to be used by practitioners for providing valid and effective results.

A systematic approach was developed to investigate the results of each operational model under consideration. The initial step was to generate sets of origin-destination volumes that represent "typical" demand flowrate patterns observed in the field. Next, the algorithmic formulations for the attained critical sum capacity and the average HCM control delay were applied to each volume scenario, based on the 6th edition formulation. Finally, statistical indications were used to assess the reliability of the critical sum method as a simplified method to predict the anticipated delay or range of delay experienced by a single-lane roundabout.

Origin-Destination Volume Combinations

The authors generated volume scenarios both systematically and with randomization built in, as shown in **Table 1**. The authors varied major and minor approach two-way volumes, splits, and turn-movement percentages. Traffic volumes ranging from 100 veh/h to 2000 veh/h were considered, ensuring that the range included both very low v/c ratio conditions as well as oversaturated conditions for a single-lane roundabout. The split factor ranges from 0.5 to 0.7, modeling both balanced and unbalanced flow conditions. Similarly, the turning-movement factor ranges from 0.05 to 0.25. Ultimately, the permutation of these analysis parameters results in 250,000 scenario combinations. Randomization is introduced to the individual volume scenarios, but the degree of randomization is always set to be less than the step size used.

Table 1 Volume scenario parameter permutations and perturbations

Direction	Parameter	Units	Min	Max	Step	Values	Randomization
EW	2 Way Volume	pceph	100	2,000	100	20	-50+[100*Rand(0,1)]
	Split percent	%	0.5	0.7	0.05	5	-0.025+[0.05*Rand(0,1)]
	Turn percent	%	0.05	0.25	0.05	5	-0.025+[0.05*Rand(0,1)]
NS	2 Way Volume	pceph	100	2,000	100	20	-50+[100*Rand(0,1)]
	Split percent	%	0.5	0.7	0.05	5	-0.025+[0.05*Rand(0,1)]
	Turn percent	%	0.05	0.25	0.05	5	-0.025+[0.05*Rand(0,1)]
Total	250,000 volume scenarios						

Analysis Methodology & Equation Documentation

The critical sum method estimates intersection demand by investigating the combination of the critical movements at the facility [12]. Essentially, the critical sum method indicates the demand volume attempting to pass through an intersection during an hour, in units of vehicles per hour per lane (veh/hr/ln). The main benefit of the critical sum method for use as a preliminary design tool is the simplicity of its calculation. Two measures of critical sum were used for the analysis, the maximum critical sum experienced by the worst-case scenario approach (CS_{MAX}), and the weighted critical sum ($CS_{weighted}$), averaging the critical sums experienced by all approaches at the roundabout, weighting the value of each approach based on the demand volume during the peak hour. The value of critical sum on each approach (CS_{i}) is found based on the calculation shown in *equation 1*, and sums the conflicting flow ($v_{c,i}$) with the approach flow ($v_{a,i}$).

$$CS_i = v_{a,i} + v_{c,i} \qquad \textit{Eq. 1}$$

$$for\ i = eastbound, northbound, westbound, southbound$$

Where:

$v_{a,i}$ = the demand flowrate on approach *i* (veh/hr)

$v_{c,i}$ = the circulating flowrate passing in front of approach *i* (veh/hr)

The maximum critical sum (CS_{MAX}) is found by selecting the highest value between the four approaches as the governing capacity for the roundabout, as shown in *equation 2*.

$$CS_{MAX} = MAX[CS_i] \qquad \textit{Eq. 2}$$

$$for\ i = eastbound, northbound, westbound, southbound$$

The weighted critical sum (CS weighted) considers the weighted average of the four approaches, as shown in *equation 3*.

$$CS_{weighted} = \frac{\sum_i (CS_i * volume_i)}{\sum_i volume_i} \qquad \textit{Eq. 3}$$

$$for\ i = eastbound, northbound, westbound, southbound$$

Since this paper determines and validates the relationship between the critical sum method and the average delay, the authors used the HCM methodology for calculating the capacity of an approach (c_i) on a single-lane roundabout with *equation 4*, and the resulting delay on that approach (d_i) with *equation 5*.

$$c_i = 1380 * exp^{(-0.00102 * v_{c,i})} \qquad \textit{Eq. 4}$$

$$d_i = \frac{3600}{c_i} + 900T\left[x_i - 1 + \sqrt{(x_i - 1)^2 + \frac{\left(\frac{3600}{c_i}\right)x_i}{450T}}\right] + 5 * min[x_i, 1] \qquad Eq.\ 5$$

$$for\ i = eastbound, northbound, westbound, southbound$$

Where:

d_i = average control delay for approach i (seconds/vehicle)

x_i = volume-to-capacity ratio of approach i (unitless)

c_i = capacity of approach i (vehicles/hour)

T = analysis time period (hour)

An example analysis for a given volume scenario is presented in **Table 2**, below.

Table 2 Sample volume scenario analysis

Input Parameters		T = 60 min			Volume Application					Critial Sum Method			HCM 6th Method			
Approach		**road vol**	**dir split**	**turn %**	**app vol**	**L**	**T**	**R**	**circ vol**	**CS**	**CS (max)**	**CS (weight)**	**cap.**	**v/c ratio**	**app delay**	**int. delay**
1	**EB**	800	0.6	0.1	480	48	384	48	359	839	839	758	957	0.50	10.0	8.58
3	**WB**				320	32	256	32	316	636			1000	0.32	6.9	
2	**SB**	700	0.55	0.15	385	58	269	58	335	720			981	0.39	8.0	
4	**NB**				315	47	221	47	490	805			837	0.38	8.8	

Statistical Indication

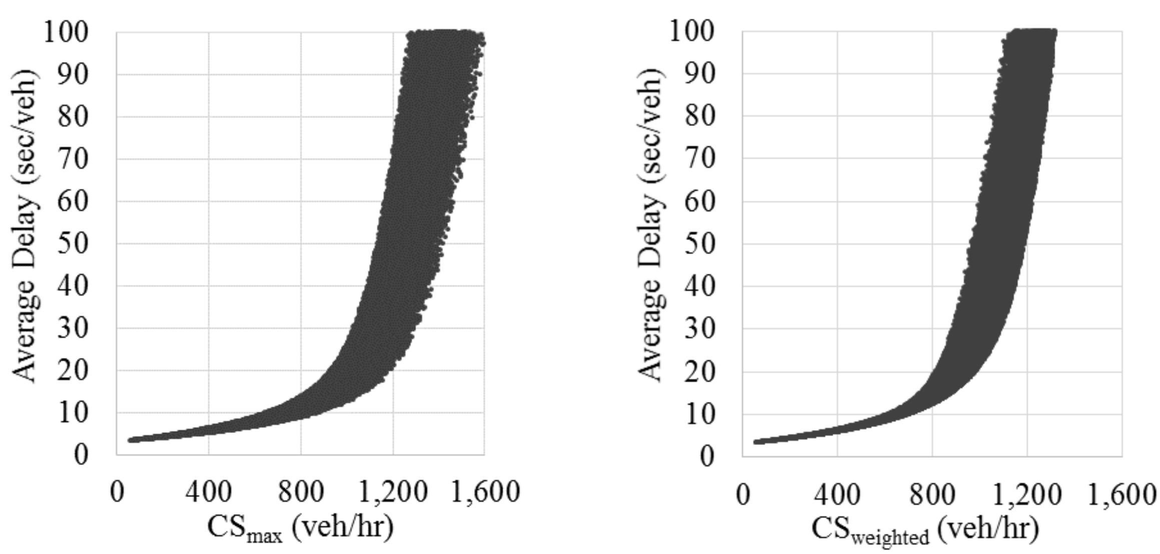
As the authors are interested in both the relationship between the critical sum and the calculated HCM delay, and the reliability of that relationship, measures are selected both of central tendency and of dispersion of the results. In this case, although median would be a good choice to avoid the influence of outlier points, the authors are more interested in the use of the mean as our measure of central tendency, being consistent with the ultimate measure of effectiveness being the average delay, and not the median delay.

Although statistical central tendency assists in visualizing how the observations are centered in a distribution, it still misses the context for interpreting the reliability of that result. Various measures of spread can be used to provide the vital information about the degree of consistency or deviation from the mean. Although standard deviation provides a statistical analysis of reliability for the data, the dataset used herein was found to not be normally distributed, nor is this measure easily translatable into practical application for consulting engineers. Based on the fact that a given LOS category for roundabouts has a spread of 10 seconds/vehicle of delay, the authors selected an acceptable range for predicting delay not to exceed ±5 seconds from the mean delay for a given critical sum value. For example, of the many volume combinations that result in a critical sum of 800, knowing the critical sum value is useful to us only if the range of delay resulting from those volume combinations is less than 10 seconds/vehicle.

Although ideally all results would fall within this acceptable range, some outliers are permissible. The authors' assessment was that if two standard deviations' worth of data, or roughly 95%, fell within the ±5 seconds/vehicle goal relative to the mean, then critical sum would be declared a robust indicator.

RESULTS OF ALGORITHM CALCULATIONS

Ideally, for the critical sum measure to be a perfect indicator variable, all volume combinations resulting in a given critical sum value would also result in a constant value for average delay. However, recognizing that this logically does not hold, the question that follows is whether the individual results deviate from the expected value within an allowable range. The volume scenarios investigated ranged from very-low volumes up to combinations well in excess of capacity for the one-lane roundabout facility. In all, 250,000 volume scenarios were run, calculating values for the maximum critical sum (CS_{MAX}), the weighted average of the critical sum ($CS_{weighted}$), and the HCM delay. While the dataset includes far more volume combinations than are likely to be encountered in the field, the authors zeroed in on a range of interest included critical sum values up to 1,600 and delay values up to 100 seconds/vehicle. While many of the points overlap and occlude information about the density of points, the range of results is shown in **Figure 2**, showing delay plotted against maximum critical sum (CS_{MAX}) on the left **(a)**, and against weighted critical sum ($CS_{weighted}$) on the right **(b)**.



(a) Maximum approach critical sum **(b)** Weighted critical sum

Figure 2 Critical sum (veh/h) versus average control delay (sec/veh)

The initial results shown in **Figure 2** indicate that volume scenarios result in a consistent relationship between critical sum and delay up until around a critical sum value of 800, representing roundabouts with Level of Service (LOS) of A or B. Using the weighted average of the critical sum value did not provide significant improvement in this result. The range of delay results gets dispersed once the critical sum exceeds 1,000 veh/hr. With this data, we can define a relationship between delay and critical sum using the mean delay values that match any given critical sum value. However, as the range in delay values increases significantly as critical sum increases past 800, further investigation is needed to examine the reliability of the average relationship. As it is the interest of the authors to investigate the reliability of planning-level analysis tools, we will be focusing further investigation on the maximum critical sum value (CS_{MAX}), and not analyzing the results of the weighted critical sum value ($CS_{weighted}$), which exhibits a significant increase in calculation complexity without providing a significant improvement in reliability as an indicator variable.

ANALYSIS OF RESULTS

The previous section provides a generalized understanding of the reliability of the critical sum method as an approach for anticipating control delay at roundabouts. However, the range of results itself does not determine the reliability, and we must also investigate the distribution of the results. In this section the authors assess the nature of the distribution of the delay values, and examine if particular sets of volume combinations have an impact on the reliability of the method.

Summarizing our experiment design, there are three input parameters for each approach, with two output variables, and we are seeking to understand how reliably the simple output variable predicts the results of the more-complex output variable. The input variables are applied to each arterial (north-south and east-west) and include total approach volume, directional split, and turn-movement percentage. In addition to these factors, each individual volume scenario has a randomization function applied to it to provide perturbation so that the full range of values is included instead of stepping each parameter. In order to summarize and analyze our results, the perturbated maximum critical sum data was binned into groups with a range of 100 vehicles/hour/lane, somewhat like stratification of the dataset. Each bin included traffic volumes within a range of -50 and +50 veh/hr. For example the critical sum bin of 1,000 represents the binned data points with critical sum values between 950 to 1,050 vehicles/lane/hour.

Maximum Critical Sum versus HCM Delay

For each set of volume scenarios resulting in a given critical sum value, a mean delay was identified as shown in **Figure 3** and **Table 3**. Additional information in Figure 3 includes the range of the individual results for delay for each volume scenario resulting in a given critical sum. Although the median result would be freer from the influence of outlier data, the mean values are supplemented with a further analysis of the distribution is conducted to establish the nature of the distribution of the results.

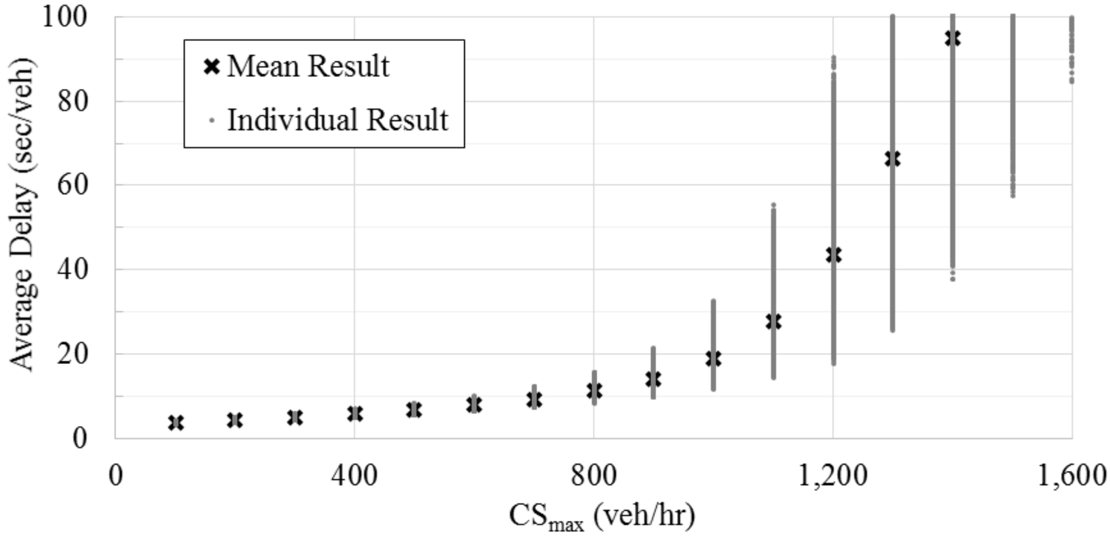
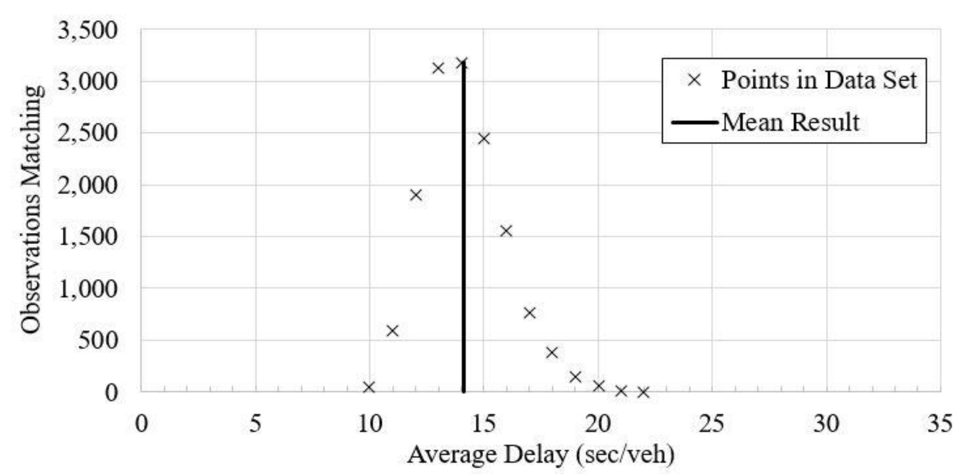


Figure 3 Maximum approach critical sum value versus average delay

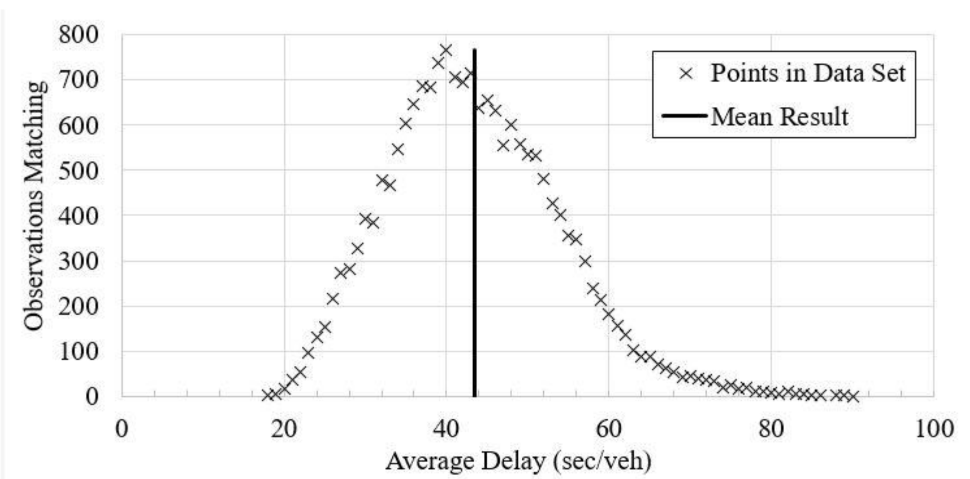
Table 3 Analysis results

Critical Sum (Max)	Mean Delay	Standard Deviation	Count of Data	Count of Data Mean ± 5 sec.	Percent of Data Mean ± 5 sec.
100	3.8	0.1	710	710	100%
200	4.3	0.2	2,389	2,389	100%
300	5.0	0.3	4,090	4,090	100%
400	5.8	0.3	5,742	5,742	100%
500	6.7	0.4	7,456	7,456	100%
600	7.9	0.6	9,108	9,108	100%
700	9.3	0.8	10,759	10,759	100%
800	11.3	1.1	12,456	12,456	100%
900	14.1	1.8	14,195	14,074	99%
1000	18.9	3.1	15,834	14,333	91%
1100	27.8	6.1	17,506	10,185	58%
1200	43.4	10.6	18,870	6,695	35%
1300	66.4	15.0	19,540	5,307	27%
1400	95.0	18.8	19,329	4,461	23%
1500	129.2	23.8	18,095	3,360	19%
1600	169.6	30.9	16,172	2,257	14%
1700	217.3	40.8	13,799	1,199	9%
1800	271.0	50.7	11,793	850	7%
1900	332.1	61.2	9,621	554	6%
2000	395.2	69.0	7,750	411	5%

The authors have chosen to examine the reliability of using this method based on the likelihood of a volume scenario producing a value of average delay that is within ±5 seconds of the calculated average delay for all volume scenarios resulting in a similar critical sum value. Referring back to the results shown on **Table 3**, reliable results from the critical sum method are seen for low-volume scenarios producing a critical sum value up to 900 veh/ln/hr. However, once the roundabout becomes congested, the percentage of delay values within the ±5 second range goes down significantly. Examining a vertical slice through the data, a plot of the number of results sharing a common critical sum and delay value is shown in **Figure 4**, including example slices along critical sum values of 900 **(a)** and 1,200 **(b)**.



(a) Sub-set of data with critical sum equal to 900 ± 50



(b) Sub-set of data with critical sum data equal to 1,200 ± 50

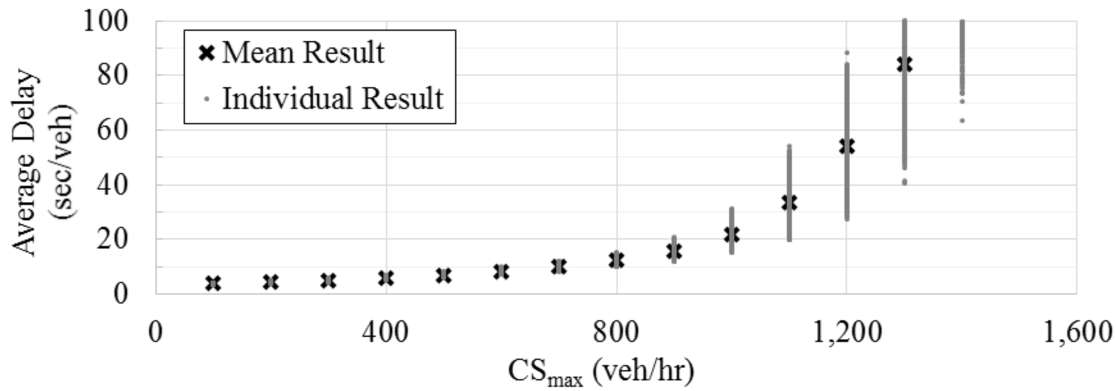
Figure 4 Distribution of average delay results relative to a given critical sum value range

Impact of Scenario Characteristics on Relationship between Critical Sum and Delay

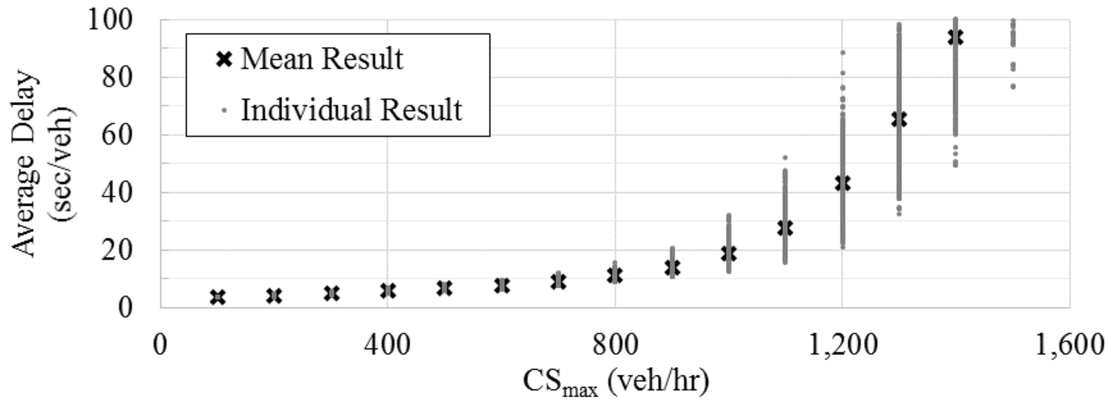
Although the general results indicate poor reliability for using the critical sum analysis as indicator for delay results, the authors wished to investigate whether the nature of the traffic flow would have an impact on overall reliability. To this end, sensitivity analysis was conducted on both the directional split and turn percentage factors to determine if their values impacted reliability.

The impact of directional split on the relationship between critical sum and delay is shown below in **Figure 5**. Three different sub-sets of the data were analyzed, including: **(a)** both roadways having balanced flow with a directional split of 0.5; **(b)** one roadway having balanced flow with a split of 0.5, and one roadway having unbalanced flow with a split of 0.7; and **(c)** both roadways having unbalanced flow with a split of 0.7. It was the hope of the authors that perhaps locations with balanced flows might prove to generate critical sum results that were more reliable for predicting delay values, but this was not found to be the case.

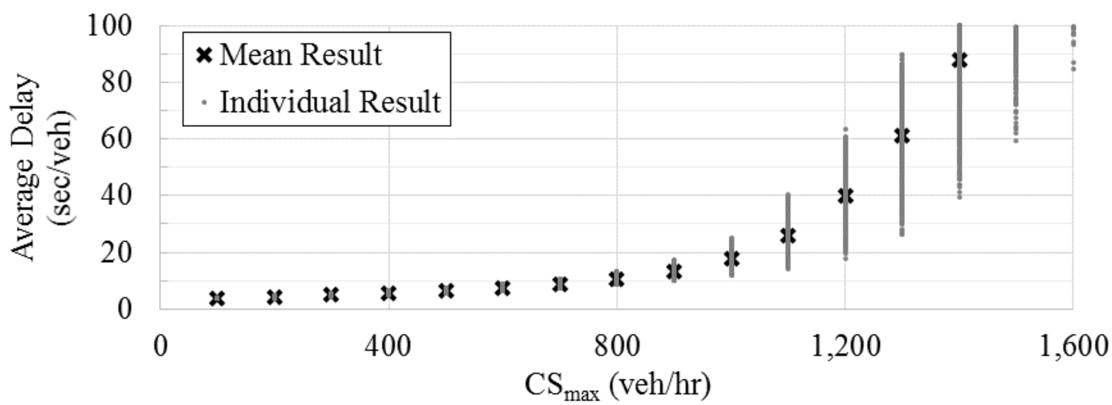
The impact of turn percentage on the relationship between critical sum and delay is shown below in **Figure 6**. Three different sub-sets of data were analyzed, including: **(a)** both roadways having low turn-movement percentages of 5%; **(b)** one roadway having a low turn-movement percentage of 5%, and one roadway having a high turn-movement percentage of 25%; and **(c)** both roadways having high turn-movement percentages of 25%. Unfortunately, as with the directional split factor, it was found that controlling the turn-movement percentage did not result in an increase in the reliability of the critical sum method as a predictor of delay.



(a) Sub-set of scenarios with directional split equal to 0.5 for both roadways

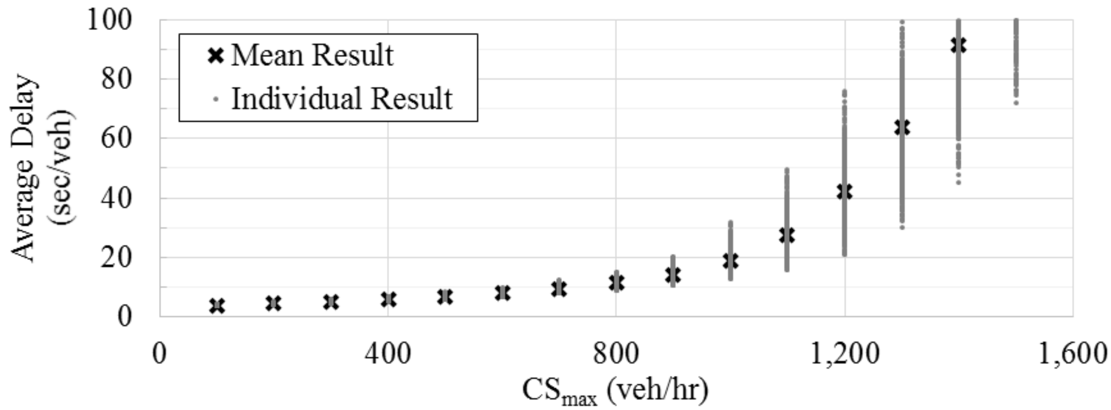


(b) Sub-set of scenarios with directional split of 0.5 for one roadway and 0.7 for the other

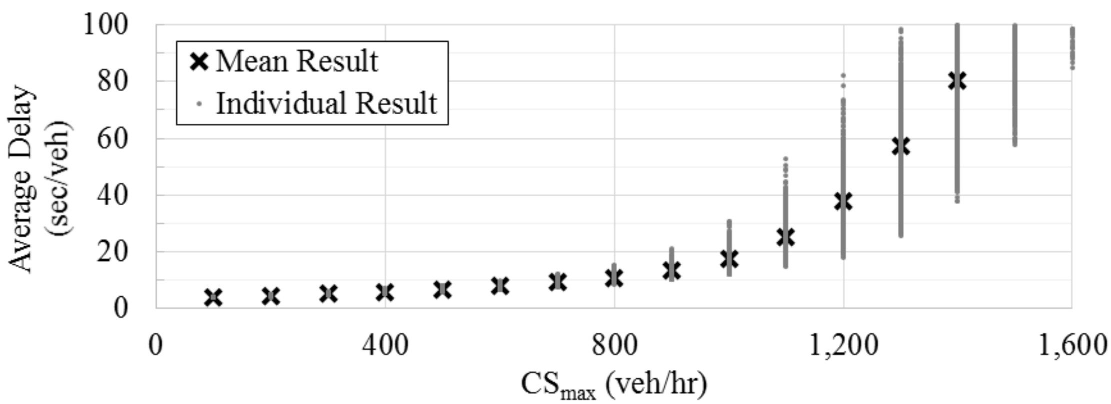


(c) Sub-set of scenarios with directional split equal to 0.7 for both roadways

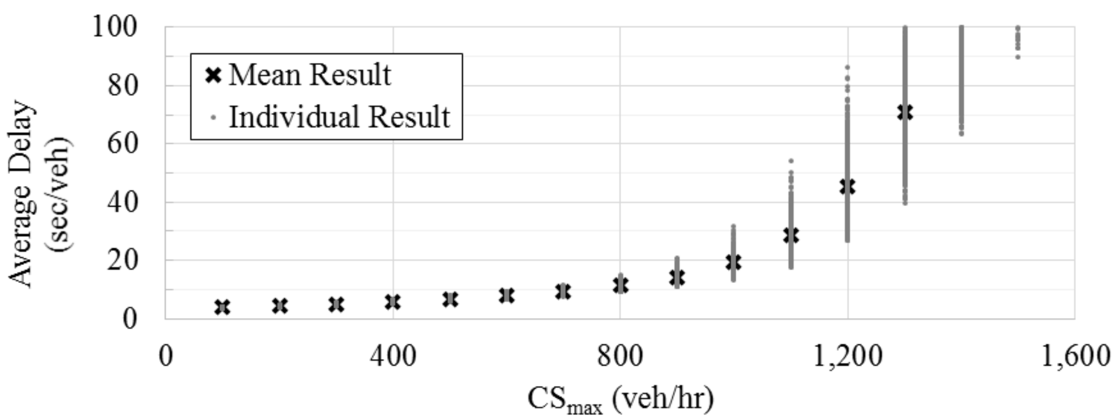
Figure 5 Impact of directional split on relationship of critical sum and delay



(a) Sub-set of scenarios with turn percentage equal to 5% for both roadways



(b) Sub-set of scenarios with turn percentage of 5% for one roadway and 025% for the other



(c) Sub-set of scenarios with turn percentage equal to 25% for both roadways

Figure 6 Impact of turn percentage on relationship of critical sum and delay

RESEARCH LIMITATIONS

The intention for using a simplified capacity-based analysis method to predict the average control delay (considered to be a benchmark,) indicating the performance of a roundabout is to assist practitioners in the preliminary planning stages of a project. Having the critical sum volumes could eliminate the necessity for conducting a detailed operational analysis which eventually saves time and effort.

In general, the exploration of two related parameters requires several things to conclude that there is a robust relationship between them. In this research, the development of a relationship between the critical sum and the average control delay contains various possible limitations. In order to have a robust relationship, all demand volume parameters need to be further explored. In reality, the directional splits and turn percentages may vary more widely than what is analyzed in the research. There might also exists several other parameters which could impact the relationship significantly, thus more volume combinations could be provide additional insight into this relationship.

CONCLUSION AND RECOMMENDATIONS

This research focused on the reliability of using critical sum algorithmic formulations, a capacity based approach, for predicting the anticipated average control delay at a single-lane roundabout. In order to be a successful planning tool for transportation practitioners, the critical sum method should be able to determine whether the performance of a roundabout (average control delay,) is close to failure, failing, or beyond failure. For assessing the effectiveness of this method, 250,000 volume scenario combinations with randomized parameters were analyzed.

To assess the reliability of the results of the simple method as a predictor for the results of the more complex method, the relationship between the two parameters was investigated. The authors plotted the maximum critical sum data points against the delay. The results showed increasing scatter in delay values of the data once the critical sum exceeded 900 veh/ln/hr. The authors further explored the option of using the weighted critical sum instead of the maximum, but it did not provide a sufficient increase in reliability to justify the additional complexity of calculation that it requires. The authors posited that the critical sum method could be considered as reliable if 95% of the delay values fell within ±5 seconds of the predicted mean value of delay for a given critical sum. It was observed from the graphs that once the critical sum exceeds 1,000 veh/ln/hr the data becomes widely spread from the mean, and does not meet the criteria for reliability.

Although the general results indicated poor reliability for using the critical sum analysis as indicator for delay results, the authors wished to investigate whether the nature of the traffic flow would have an impact on overall reliability. To this end, sensitivity analysis was conducted on both the directional split and turn percentage factors to determine if their values impacted reliability. Unfortunately, controlling these factors was not found to have a significant improvement on the reliability of the critical sum method.

Ultimately, using the critical sum method confidently anticipates whether the Level of Service (LOS) for a roundabout is an A or a B based on the HCM average delay standards for un-signalized intersections, but does not provide reliable results when they are most needed, attempting to determine if the facility will operate as D, E, or F. Thus, the authors conclude that not enough evidence was found from the data which indicates the reliability of the critical sum method in anticipating the average control delay for a single-lane roundabout. As the critical sum method is currently in use as a planning tool for other intersection facility types, the authors recommend further studies be performed regarding the reliability and efficiency of using the critical sum method as an accurate delay predictor for these other facilities. Identifying such information would provide a research-based validation for which of these other intersection designs could accurately be represented by the critical sum method.

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